Part III: Experimental Applications of a Practical Model for High-Gradient Magnetic Separation to Pilot-Scale Coal Beneficiation

The computer implementation and experimental verification of the practical model for high-gradient magnetic separation (HGMS) based on particle buildup and feed characteristic, developed in Part II, are described. It is shown that the new model can be used to satisfactorily predict the grade, recovery and concentration breakthrough observed in pilot-scale experimental studies of HGMS for the removal of sulfur and ash from water slurries of several pulverized Eastern coals. The experimentally verified model can be used to quantitatively identify the tradeoff of separation variables so as to optimize the magnetic removal of sulfur and ash from pulverized coal. The model can also be used to assess the technical and economical feasibility of the magnetic beneficiation of pulverized coal without extensive experimental tests.

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SCOPE

In Part II, the important factors of particle capture and buildup, and feed characteristic in determining the technical performance of HGMS have been quantitatively examined. It was suggested that none of the existing mathematical models could be used to quantitatively predict the grade and recovery of the separated product and the capacity (concentration breakthrough) of the separator observed in pilot-scale experimental studies of HGMS. A new practical model based on particle buildup and feed characteristic for quantitatively predicting the technical performance of HGMS was then proposed and illustrated with simple applications.

The present work is concerned with the computer implementation and experimental verification of the new model, with particular emphasis on an important and promising application of HGMS, namely, the magnetic removal of inorganic sulfur and ash from pulverized coal. Previous experimental studies have indicated clearly that most of the mineral impurities in coal, which contribute to its pyritic sulfur, sulfate sulfur and ash contents, are weakly magnetic (paramagnetic). Those sulfur-bearing and ash-forming minerals, if sufficiently liberated as discrete particles, can be separated normally from the pulverized coal which is practically nonmagnetic (diamagnetic) (Ergun and Bean, 1968; Liu and Lin, 1976).

Indeed, HGMS was first successfully adapted in 1973 in a bench-scale experimental study to remove sulfur and ash from a pulverized Brazilian coal suspended in water (Trindade, 1973). Since then, the technical feasibility of utilizing cyclic and continuous HGMS units for the magnetic separation of sulfur and ash from water slurries of pulverized U.S. coals has been demonstrated in a number of experimental studies, with substantial amounts of sulfur and ash removal being achieved. In addition, studies on the quantitative modeling and correlation of beneficiation performance as well as the conceptual design and cost estimation of HGMS processes for beneficiation of coal/water slurries have been reported. A review of published literature on HGMS applied to coal beneficiation can be found in Liu (1982).

Specifically, in this work, a pilot-scale experimental study of HGMS applied to the beneficiation of water slurries of a pulverized Illinois No. 6 coal and several pulverized Pennsylvania coals has been conducted over a wide range of separation conditions. The experimentally observed grade, recovery and concentration breakthrough are then compared with the predictions of the new model. The applicability of the model to quantitative predict the experimentally observed technical performance of pilot-scale HGMS is evaluated. The key features of a general computer program for implementing the new model are also discussed.

CONCLUSIONS AND SIGNIFICANCE

Based on the agreement between model predictions and experimental results presented in this work and the successful applications of the model to other pulverized coals described elsewhere (Oak, 1977), it can be suggested that there is now an experimentally verified, practical model for predicting the grade, recovery and concentration breakthrough observed in pilot-scale HGMS for the beneficiation of water slurries of pulverized coal prior to actual separation testing. The model requires only the knowledge of the separation conditions and feed characteristic. The latter can be readily determined from the particle-size measurement of the feed stream and from simple analyses of samples obtained from a standard float-and-sink separation of the feed stream. A general computer program for implementing the new model has also been successfully developed and tested.

The experimentally verified, new model can be used to quantitatively identify the tradeoff of separation variables such as field intensity, slurry velocity and particle size so as to optimize the removal of sulfur and ash from water slurries of pulverized coals by HGMS. The model can also be used to assess the technical and economical feasibility of the magnetic beneficiation of coal without extensive experimental tests.

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PREVIOUS WORK ON CORRELATIONS OF PERFORMANCE OF HGMS APPLIED TO COAL BENEFICIATION

Qualitative Correlation Using Force Balance Model

The first qualitative model for correlating the performance of HGMS applied to coal beneficiation was the force balance model due to Trindade (1973). The model postulates a simplified picture of an HGMS unit as consisting of an isolated strand of cylindrical ferromagnetic stainless-steel wool of a uniform cross-section inserted horizontally in the bore of a solenoidal magnet where the magnetic field is uniform vertically. The pyrite particles of various sizes are carried in a water slurry flowing downward by gravity past the magnetized steel wool strand. The net vertical force, F_{net}, acting on a given particle located at a certain point close to the magnetized strand results from the balance of the following forces: (i) F_{mv}—the vertical (upward) component of the magnetic force attracting the particle toward the strand; (ii) F_d—the hydrodynamic drag force pushing the particle downward; and (iii) W—the net weight of the particle, i.e., weight minus buoyance, also pointing downward. Consequently,

$$F_{net} = F_{mv} - (W + F_d) \tag{1a}$$

$$R = |F_{mv}/(W + F_d)| \tag{1b}$$

where R is the ratio between the magnetic and competing forces. The model considers the value of the force ratio R as a measure of the probability of capture of the particles, and suggests that for an effective separation the ratio R must be of the order or greater than unity, i.e., $R \geq 1$. A series of model simulations were carried out by Trindade to examine the effect of particle size on the force ratio R for different slurry velocities. The results showed that there should be a certain particle size for which R would reach a maximum, and R would decrease with increasing slurry velocity. Therefore, one would expect that the sulfur concentration and recovery in the magnetic refuse (mags) would peak at a certain particle size at a given slurry velocity, if pyrite is the dominant form of sulfur and it is sufficiently liberated. These observations were confirmed by experimental data from HGMS tests on water slurries of pulverized Brazilian Siderpolis coals of narrowly distributed (approximately monodispersed) particle sizes. The simple force balance model formulated by Trindade has provided a qualitative basis for correlating the grade and recovery of the magnetically cleaned coal achieved by HGMS for a variety of combinations of separation variables. The model, however, cannot be used to correlate the separator capacity (concentration breakthrough) under given separation conditions.

Quantitative Correlations Using Particle Capture (Trajectory) and Buildup Models

A number of recently developed models based on the theoretical analysis of particle capture (trajectory) and buildup have been applied to quantitatively correlate the grade and/or recovery of the magnetically clean coal from HGMS of water slurries of pulverized coals. The first model applied was the particle trajectory model presented by Bean (1971). This model was intended as an approximation of the gross characteristics of the ideal or unloaded magnetic separator performance without requiring sophisticated calculations. In the model, the slurry with an inlet velocity V_o (which is directly proportional to the reciprocal of residence time τ) and a viscosity η is assumed to be moving along the magnetic field of an intensity H_o at a high Reynolds number relative to an array of ferromagnetic collecting cylinders of a radius a and a saturation magnetization M. The cylinders occupy a void volume fraction ϵ (one minus the packing density factor, F) of the solenoidal volume (magnetic separator matrix) of a length L. For particles of a magnetic susceptibility χ and a radius R, the particle concentrations at the inlet and outlet of the separator matrix, C_{in} and C_{out} respectively, can be related by the following expression:

$$\frac{C_{out}}{C_{in}} = \exp\left[-\frac{4\pi}{3}\cdot\frac{(\mu MH)(\chi R^2)}{(\eta V_0/\epsilon)}\cdot\frac{(1-\epsilon)L}{\pi a^2}\right] \tag{2}$$

The model implies that a plot of the logarithm of the ratio C_{out}/C_{in} vs. the parameter $HL\epsilon(1-\epsilon)/V_o$ or $H\tau\epsilon(1-\epsilon)$ should yield a straight line, if other parameters included in this model are all kept constant and the experiment data can be satisfactorily correlated by the model. Bean's model was successfully applied in the authors' laboratory to quantitatively correlate the data from HGMS of water slurries of a pulverized Illinois No. 6 coal over a wide range of field intensity, residence time and packing void volume fraction at a slurry concentration of 2.7 wt. % (Lin et al., 1976). The experimental verification of the Bean's model allows one to quantitatively identify the tradeoff of operating variables, such as field intensity H, residence time τ and packing void volume fraction ϵ, so as to optimize the magnetic separation of sulfur and ash. Note that the theoretical analysis presented in Part II has shown that both particle capture (trajectory) and buildup depend highly on the ratio of the magnetic velocity V_m of the particle to the free stream fluid velocity V_∞. The latter is defined by the inlet fluid velocity V_o divided by the packing void volume fraction ϵ. The magnetic velocity V_m is essentially a terminal velocity of a particle in a magnetic field, which is defined such that

$$\frac{V_m}{V_\infty} = \frac{2}{9}\cdot\frac{(\mu MH)(\chi R^2)}{(\eta V_0/\epsilon)}\cdot\frac{1}{a} \tag{3}$$

The ratio V_m/V_∞ contains almost all the major independent variables in HGMS: (1) magnetic field terms, μMH-magnetization of the particle collecting wire M and field intensity H; (2) particle property term, χR^2-particle magnetic susceptibility χ and radius R; (3) flow field term, $\eta V_o/\epsilon$-fluid viscosity η, inlet velosity V_o and packing void volume fraction ϵ; and (4) matrix packing term, $1/a$ where a is the wire radius. It is of interest to note that the first three terms appear explicitly in the exponent of Bean's model, and the fourth (matrix packing) term, $1/a$, is replaced by $(1-\epsilon)L/\pi a^2$ in the exponent of Bean's model.

A particle trajectory model developed by Stekly and Minervini (1976) has been applied to quantitatively correlate the performance of HGMS applied to coal beneficiation (Stekly et al., 1980). This model relates the particle concentrations at the inlet and outlet of the separator matrix, C_{in} and C_{out}, respectively, by an exponential function of major independent variables. Depending upon the magnitude of the ratio V_m/V_∞, the model has the following two forms:

$$\frac{C_{out}}{C_{in}} = \begin{cases} \exp\left[-\frac{2}{9}\cdot\frac{(\eta MH)(\chi R^2)}{(\eta V_o/\epsilon)}\cdot\frac{(1-\epsilon)L}{\pi a^2}\right] & \text{(for small } V_m/V_\infty) \\ \exp\left[\frac{-3^{5/6}}{2^{2/3}}\cdot\frac{(\mu MH)^{1/3}(\chi R^2)^{1/3}a^{2/3}}{(\eta Vo/\epsilon)^{1/3}}\cdot\frac{(1-\epsilon)L}{\pi a^2}G\right] & \text{(for large } V_m/V_\infty) \end{cases} \tag{4}$$

In the model, G is an empirical constant representing the degree of matrix loading which must be fitted by using experimental data. This model suggests that for large values of V_m/V_∞, a plot of logarithm of the ratio (C_{out}/C_{in}) versus the parameter $(H/V_o)^{1/3}L$ should yield a straight line if other parameters included in the model are kept constant and the experimental data can be satisfactorily correlated by the model. The model was successfully applied to quantitatively correlate the data obtained from HGMS of water slurries of a pulverized Pennsylvania Upper Freeport (Delmont) coal over a wide range of field intensity, slurry velocity and matrix length at a slurry concentration of 20 wt. % (Stekly et al., 1980).

Another particle trajectory and buildup model developed by Luborsky and Drummond (1975, 1976) has also been applied to quantitatively correlate the experimental data obtained from HGMS of water slurries of a pulverized Pennsylvania Upper Freeport (Delmont) coal (Luborsky, 1978). This model assumes the use of ribbon-like particle collecting wires, instead of cylindrical ones as in the previous models. It considers the possibility of mechanical entrapment of particles in the separator matrix. The model also relates the particle concentrations at the inlet and outlet of the separator matrix, C_{in} and C_{out}, respectively, by an exponential function. The exponent of the model includes two empirical constants which represent the fraction of the particle collecting wires that are active in capturing particles (i.e., the capture efficiency), and the fraction of the particles that are mechanically entrapped. Both empirical constants must be determined by properly fitting the experimental data. A detailed description of the model and its experimental application to HGMS of water slurries of pulverized coals can be found in Luborsky (1978), in which it is shown that the model calculations can be made to fit the experimental data only approximately.

The preceding particle trajectory and buildup models for HGMS provide a useful means for quantitative correlations of the effects of separation variables on the grade and/or recovery of the beneficiated coal. However, these models cannot be used to quantitatively correlate or predict such effects based on the characteristic of the pulverized coal prior to actual magnetic separating testing. In addition, it is important to recognize that none of these models can be used to quantitatively predict the separator capacity or concentration breakthrough in the beneficiation of coal by HGMS. In what follows, the experimental applications of the practical model of Part II to quantitatively predict the grade, recovery and concentration breakthrough of HGMS applied to pilot-scale coal beneficiation are described.

EXPERIMENTAL WORK

Materials and Analyses

The coals used in this study included an Illinois No. 6 coal from Perry County, and three Pennsylvania coals from Jefferson County, namely, Upper Freeport, Lower Freeport and Lower Kittanning seams. Standard float-and-sink separation was used to classify the pulverized coal sample into different density fractions. Tetrabromomethane, carbon tetrachloride and benzene having densities of 2.96, 1.60 and 0.879 gm/cm^3, respectively, were used as the separating agents. The magnetic susceptibility of each separated fraction was determined by the Guoy balance method. The combustion iodometric procedure using a Leco sulfur analyzer (Model 532) was employed to analyze the total sulfur content. The accuracy of the Leco sulfur analyzer was about ±0.1% for samples containing over 2.0 wt. % total sulfur, and ±0.5% for samples containing less than 20 wt. % total sulfur. The standard ASTM procedures D271-70 and D2492-68 were used for analysis of the ash and pyritic sulfur contents, respectively. Also, the particle size of the pulverized feed coal sample was measured by a Coulter Counter (Model TA).

Experimental System and Procedure

A schematic diagram of the pilot-scale HGMS system for the experiments of magnetic beneficiation of coal/water slurry is shown in Figure 1. The water slurry of coal was stored in a 0.114-m^3 (30-gal) feed tank mounted with a Lightnin mixer. A Hills-McCanna constant volume pump with a maximum capacity of 9.78 m^3/s (1.55 gal/min) and a maximum discharge pressure of 1.138 MPa (165 psi) was used to pump the slurry to the high-intensity, high-gradient magnetic separator. The magnetic separator used was a pilot-plant model manufactured by Pacific Electric Motor Co. Data from pilot-plant tests in kaolin beneficiation and water treatment with the same type of separator have been used for direct scale-up to large commercial units. The maximum field intensity which can be generated by this separator is 2 Tesla (1.591 MA/m or 20 kOe) in an open volume of 0.129 m in side diameter and 0.533 m in length. A separator matrix filled with compressed pads of stainless-steel wool was placed in the open volume, and the coal slurry to be magnetically beneficiated was pumped through the matrix. The experiments were performed with matrices of two different sizes. The first one was made from a 2.54-cm schedule-40 stainless-steel pipe of 2.66-cm inside diameter and 50.8 cm in length. The second one was constructed from a 50.8-cm long stainless-steel tube of 12.7-cm inside diameter. The packing material used was medium-sized 430 stainless-steel wool with an average diameter of about 100 μm.

To conduct the wet beneficiation experiments, a coal slurry of a fixed concentration was prepared by mixing known amounts of coal, water and wetting agent (Alconox). Possible air bubbles trapped in the system were first eliminated by passing clean water through the system at a high flow velocity. The feed slurry was then thoroughly agitated by a stirrer, and

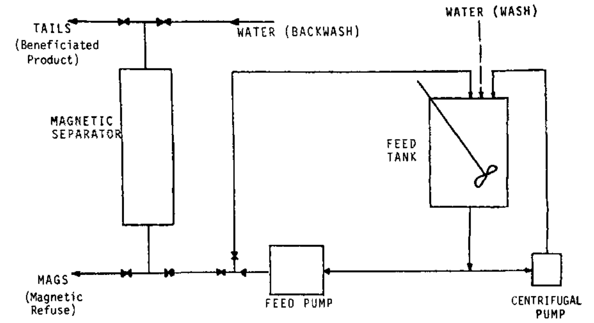

Figure 1. A schematic diagram of the pilot-scale HGMS system for coal beneficiation experiments.

circulated at the desired experimental flow velocity within the closed loop between the outlet of the feed pump and the inlet to the feed tank. After a steady state has been reached within this closed loop in a period of 30 minutes to 3 hours, the well-mixed coal slurry was pumped through the matrix of the magnetic separator of known packing density at fixed flow rate and field intensity. Time-cut samples of the beneficiated product (tails) passing through the separator matrix were collected and analyzed for ash, total sulfur and pyritic sulfur contents. The feed slurry was also sampled and analyzed.

EXPERIMENTAL RESULTS AND MODEL PREDICTIONS

Experimental Verification and Application of the Practical HGMS Model: Pulverized Illinois No. 6 Coal

Float-and-Sink Separation and Average Magnetic Susceptibility Distribution. The float-and-sink technique separates coal particles into fractions of different densities. The heavier fractions consist mainly of mineral impurities, and the lighter fractions contain mostly pure coal. The average magnetic susceptibility of coal particles in each separated fraction depends on the constituents and amounts of mineral particles. Table 1 illustrates the results of the float-and-sink separation of the pulverized Illinois No. 6 coal. Also included in the table are the average density and magnetic susceptibility along with the ash, total sulfur and pyritic contents of each separated fraction.

In Figure 2a, the measured values of average volume magnetic susceptibility (χ) are plotted against the cumulative wt. % (W) of the pulverized Illinois No. 6 coal particles. The data can be satisfactorily approximated linearly for the three fractions with the cumulative wt. % ranging from 0 to 85, 85 to 95, and 95 to 100:

$$\chi(W) \times 10^6 \text{ emu/cm}^3 = \begin{cases} 3.02 \times 10^{-2}W - 0.231, & 0 \le W < 85 \\ 0.347W - 27.18 \times 10^1, & 85 \le W < 95 \\ 2.491 \times 10^1 W - 2.360 \times 10^3, & 95 \le W \le 100 \end{cases} \quad (5)$$

Figure 2a shows that there is a sharp increase in the average magnetic susceptibility of the pulverized Illinois No. 6 coal particles at a cumulative wt. % of 95. This result suggests that if a magnetic separator could remove all the liberated magnetic components from the pulverized Illinois No. 6 coal, the maximum recovery of the beneficiated coal could be about 95 wt. %. Indeed, the latter was found to be the maximum recovery observed in the recent pilot-scale experimental study of HGMS applied to the beneficiation of the same pulverized coal suspended in a water slurry conducted in the authors' laboratory (Lin et al., 1976).

Float-and-Sink Separation and Ash, Total Sulfur and Pyritic Sulfur Distributions. The ash, total sulfur and pyritic sulfur contents of each separated fraction obtained from the float-and-sink analysis of the pulverized Illinois No. 6 coal are plotted against the cumulative wt. % (W) of coal particles in Figure 2b. As in Figure 2a, the measured ash, total sulfur and pyritic sulfur distributions depicted in Figure 2b can be satisfactorily approximated by several linear sections. In particular, the ash, total sulfur and pyritic sulfur contents of the pulverized Illinois No. 6 coal at a cumulative wt. % W, denoted by $A_1(W)$, $A_2(W)$ and $A_3(W)$, respectively, can be written as:

$$\underset{(\text{Ash, wt. \%})}{A_1(W)} = \begin{cases} 0.164W + 1.284, & 0 \le W < 85 \\ 1.878W - 144.382, & 85 \le W < 95 \\ 4.542W - 397.47, & 95 \le W \le 100 \end{cases} \quad (6)$$

TABLE 1. FLOAT-AND-SINK CHARACTERIZATION OF PULVERIZED ILLINOIS NO. 6 AND PENNSYLVANIA UPPER FREEPORT COALS USED

Density (g/cm³)	Average Density (g/cm³)	Wt. %	Avg. Magnetic Susceptibility (emu/g)	Avg. Magnetic Susceptibility (emu/cm³)	Ash (Wt. %)	Total Sulfur (WT. %)	Pyritic Sulfur (Wt. %)
			(Illinois No. 6 Coal)				
−1.257	1.250	1.11	−0.191	-0.23875×10^{-2}	1.42	2.10	0.10
+1.257−1.310	1.284	10.18	−0.062	-7.98648×10^{-2}	2.46	2.32	0.16
+1.310−1.357	1.334	55.50	0.798	1.06453	7.33	2.61	0.47
+1.357−1.409	1.383	23.32	1.50	2.07312	14.29	2.67	0.93
+1.409−1.462	1.436	5.63	3.31	4.74598	28.28	4.35	2.66
+1.462−1.590	1.526	3.24	17.50	26.7355	41.22	7.80	6.76
+1.590	1.670	2.02	63.00	105.193	52.18	16.94	15.93
Average					11.51	3.14	1.18
			(Pennsylvania Upper Freeport Coal)				
−1.251	1.240	2.40	0.385	0.477	1.91	0.53	0.11
+1.251−1.318	1.285	65.14	0.459	0.590	2.92	0.68	0.17
+1.318−1.374	1.346	22.07	1.000	1.346	6.38	0.84	0.43
+1.374−1.442	1.408	4.10	2.240	3.154	15.87	1.90	1.43
+1.442−1.515	1.479	1.84	2.780	4.112	22.41	3.23	2.41
+1.515−1.588	1.552	1.37	4.124	6.400	35.07	7.42	5.90
+1.588	1.650	3.08	7.156	11.807	57.05	28.26	21.86
Average			0.949	1.250	7.7	1.83	1.15

$$A_2(W) \ (\text{Total sulfur wt. \%}) = \begin{cases} 6.688 \times 10^{-3}W + 2.218, & 0 \le W < 85 \\ 1.878W - 144.382, & 85 \le W < 95 \\ 4.542W - 397.474, & 95 \le W \le 100 \end{cases} \quad (7)$$

$$A_3(W) \ (\text{Pyritic sulfur wt. \%}) = \begin{cases} 1.064 \times 10^{-2}W + 0.086, & 0 \le W < 85 \\ 0.241W - 19.507, & 85 \le W < 95 \\ 3.140W - 294.895, & 95 \le W \le 100 \end{cases} \quad (8)$$

Figure 2b shows that the ash and pyritic sulfur contents of the Illinois No. 6 coal increases with increasing cumulative weight percent of coal particles. This observation is similar to that shown in Figure 2a, in which the average magnetic susceptibility also increases with increasing cumulative weight percent. Figure 2b further shows that the sulfur content of the pulverized Illinois No. 6 coal particles remains essentially constant up to a cumulative wt. % of 85. This figure suggests that if the particle sizes of the pulverized Illinois No. 6 coal are approximately monodispersed or narrowly distributed and if the recovery of the beneficiated coal is smaller than 85 wt. %, the expected magnetic removal of sulfur will be essentially constant and independent of the separation conditions. Indeed, the latter observation has been confirmed by experimental results obtained for the same pulverized coal from a recent pilot-scale study conducted in the authors' laboratory (Lin et al., 1976). For example, Figure 3 and Table 6 in Lin et al. (1976) show that for experiments under relatively similar separation conditions which resulted in a recovery of the beneficiated Illinois No. 6 coal below 85 wt. %, the changes in the sulfur content of the beneficiated coal with increasing retention time were not appreciable.

The preceding discussion in connection with Figures 2a and 2b clearly suggests that the proposed modeling framework described in Part II allows one to obtain the proper conceptual understanding of the technical performance of HGMS applied to coal beneficiation. In particular, the proposed method for characterizing the feed coal by a standard float-and-sink separation and measurements of the magnetic susceptibility along with the ash, total sulfur and pyritic sulfur contents of each separated fraction, as illustrated by the experimental results obtained for the Illinois No. 6 coal, provides some indication about the expected grade and recovery of the beneficiated coal prior to actual magnetic separation testing.

Particle-Size Distribution. The particle-size distribution of the pulverized Illinois No. 6 coal was measured by a Coulter Counter in the size range of 6 to 160 micron. The results were correlated by the Rosin-Rammler equation, which is widely employed for characterizing the particle-size

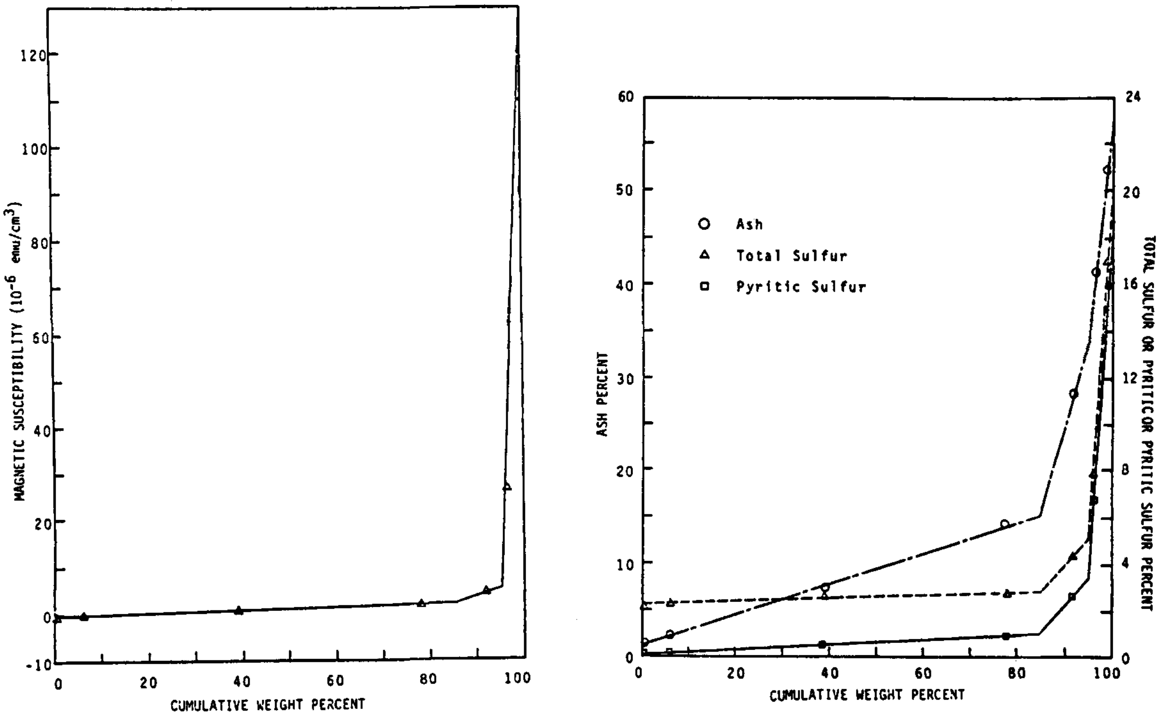

Figure 2. Characterization of the pulverized Illinois No. 6 coal by (a) the magnetic susceptibility distribution, and (b) the ash, total sulfur and pyritic sulfur distributions.

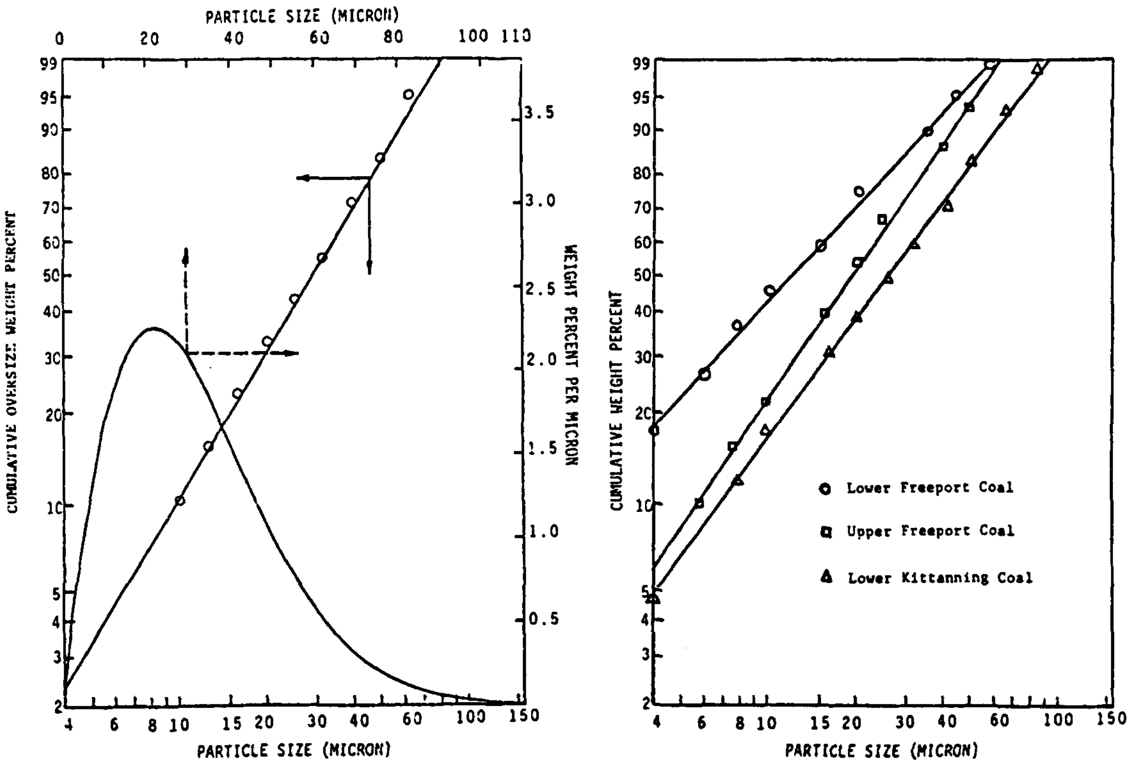

Figure 3. Particle size distributions of (a) the pulverized Illinois No. 6 coal (left) and (b) the pulverized Pennsylvania Lower Freeport, Upper Freeport and Lower Kittanning coals (right).

distributions in coal processing operations (Leonard, 1979):

$$O(D_p) = 100 - U(D_p) = 100 \exp\left[-\left(\frac{D_p}{m}\right)^n\right] \quad (9)$$

In Eq. 9, $O(D_p)$ is the cumulative weight percent of oversized particles of a minimum diameter D_p, $U(D_p)$ is the cumulative weight percent of undersized particles of a maximum diameter D_p, and m and n are size distribution constants. Figure 3a illustrates a log-log plot of the measured cumulative oversize weight percent distribution of the pulverized Illinois No. 6 coal. The linear distribution observed in the log-log plot suggests that the Rosin-Rammler equation satisfactorily correlates the experimental data. Also included in Figure 3a is the measured differential weight percent distribution drawn on a regular scale. The latter shows that most of the particles of the pulverized Illinois No. 6 coal have sizes ranging from 5 to 50 micron.

Magnetic Velocity Distributions and Computer Model Implementation. The magnetic velocity distributions for coal, ash, total sulfur and pyritic sulfur particles for characterizing the feed stream at given separation conditions can be determined by numerically integrating Eqs. 34–35 of Part II based on the experimentally measured distributions of magnetic susceptibility and particle size and density, along with ash, total sulfur and pyritic sulfur contents of the pulverized Illinois No. 6 coal used. As defined in Part II, the magnetic velocity distribution for the pulverized feed coal stream, $F(V_m)$, corresponds to the total weight fraction of magnetic particles in the feed stream having a magnetic velocity less than V_m. By introducing the cumulative undersize distribution function, $U(D_p)$ given by Eq. 9, an explicit equation for $F(V_m)$ can be written from Eq. 34 of Part II and Eq. 9 as:

$$F(V_m) = \frac{1}{100}\int_{W_o}^{100} U(D_p)dW + \frac{W_o}{100}$$

$$= \int_{W_o}^{100}\left\{1 - \exp\left[-\left(\frac{D_p}{m}\right)^n\right]\right\} dW + \frac{W_o}{100} \quad (10)$$

Likewise, the magnetic velocity distributions of ash, total sulfur and pyritic sulfur (i = 1, 2 and 3, respectively), denoted by $F_i(V_m)$, can be written from Eq. 35 of Part II as:

$$F_i(V_m) = \frac{1}{T_i}\int_{W_o}^{100} \frac{A_i(W)}{100}\left\{1 - \exp\left[-\left(\frac{D_p}{m}\right)^n\right]\right\} dW + \frac{A_{io}W_o}{100} \quad (11)$$

Here, T_i, i = 1, 2, and 3 are the average ash, total sulfur and pyritic sulfur contents, respectively, of the pulverized feed coal (for example, Table 1): $A_i(W)$, i = 1, 2 and 3, are the ash, total sulfur and pyritic sulfur distributions, respectively, of the pulverized feed coal (for example, Eqs. 6 to 8 and Figure 2b); and A_{io}, i = 1, 2 and 3, are the average ash, total sulfur and pyritic sulfur contents, respectively, of the fraction W_o with negative magnetic susceptibilities, Table 1. Note that the particle diameter D_p in both Eqs. 10 and 11 is an implicit function of W. This can be seen from the dependence of the particle radius R on the magnetic susceptibility distribution $\chi(W)$ (for example, Eq. 5 and Figure 2a) given by Eq. 37 of Part II:

$$D_P = 2R$$

$$= 2\left[\frac{9\eta a}{9\mu_o M H_o \chi(W)}\right]^{1/2} V_m^{1/2} \quad (12)$$

Figure 4 illustrates the magnetic velocity distributions of the feed coal, ash,

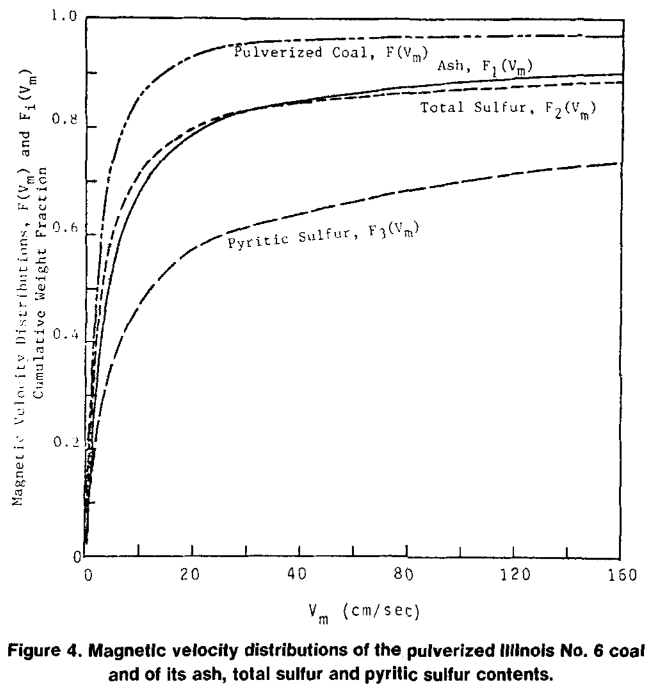

Figure 4. Magnetic velocity distributions of the pulverized Illinois No. 6 coal and of its ash, total sulfur and pyritic sulfur contents.

total sulfur and pyritic sulfur obtained by substituting Eq. 12 into Eq. 10 and 11 and integrating the latter equations for $F(V_m)$ and $F_i(V_m)$ numerically based on the experimental data in Table 1 and Figures 2a to 3 along with the corresponding correlations Eqs. 5 to 8 under the following conditions: $\eta = 10^{-3}$ Pa·s (1 cp), $a = 30 \times 10^{-6}$m, $\mu_o = 4\pi \times 10^{-7}$ Henry/m and $M = 0.14$ Tesla (0.111 MA/m or 1.4 kOe). For separations at other conditions, feed streams can be characterized by the magnetic velocity distributions calculated in a similar fashion.

When V_m is equal to the minimum magnetic velocity for particle buildup, $V_{m,\min}$, as defined by Eq. 30 of Part II, $1 - F(V_m)$ represents the total weight fraction of magnetic particles in the feed stream which will be captured inside the separator matrix. The corresponding minimum particle diameter, denoted by $D_{p,\min}$, can be related to $V_{m,\min}$ and to the total weight of the magnetic particles captured per unit cross section of the separator matrix, (LOAD), by substituting Eq. 30 of Particle II for $V_{m,\min}$ into Eq. 12 to give the following expression:

$$D_{p,\min} = 2\left[\frac{9\eta a}{9\mu_o M H_o \chi(W)}\right]^{1/2} V_{m,\min}^{1/2}$$

$$= 2\left[\frac{9\eta a}{9\mu_o M H_o \chi(W)}\right]^{1/2}\left[V_\infty\left\{\frac{(\mathrm{LOAD})}{2LFd} + 1/4\right\}^{3/2}(4.45)\right]^{1/2}$$

$$= \left[\frac{80.1\eta a V_\infty}{\mu_o M H_o}\right]^{1/2}\left[\frac{(\mathrm{LOAD})}{2LFd} + 1/4\right]^{3/4}[\chi(W)]^{-1/2} \qquad (13)$$

In Eq. 13, the only unknown variable prior to actual magnetic separation tests is (LOAD), the total weight of magnetic particles captured by a unit cross section of the separator matrix. This quantity can be predicted from the total weight of magnetic particles fed to a unit cross-section of the separator matrix, denoted by (FEED), by integrating Eq. 32 of Part II together with Eqs. 10 and 13 as follows:

$$\frac{d(\mathrm{LOAD})}{d(\mathrm{FEED})} = 1 - F(V_m), \qquad V_m = V_{m,\min} \qquad (14)$$

$$= 1 - \int_{W_o}^{100}\left\{1 - \exp\left[-\left(\frac{D_p}{m}\right)^n\right]\right\}dW + \frac{W_o}{100}, \quad D_p = D_{p,\min} \qquad (15)$$

$$= 1 - \int_{W_o}^{100}\left\{1 - \exp\left[-\left(\frac{D_{p,\min}}{m}\right)^n\right]\right\}dW + \frac{W_o}{100} \qquad (16)$$

The last equation can be numerically integrated first with respect to W by substituting Eq. 13 for $D_{p,\min}$, which is a function of the cumulative weight percent W through the term $[\chi(W)]^{-1/2}$. The integrated result represents a first-order ordinary differential equation with respect to the variable, (LOAD). It can be readily integrated further by standard numerical methods to give the amount of magnetic particles captured, (LOAD), as a function of the total feed amount, (FEED), per unit cross-section of the separator matrix. Note that essentially the same integration procedure can be used to find the amounts of specific magnetic particles of interest, such as ash, total sulfur and pyritic sulfur, as explicit functions of their total feed amounts per unit cross-section of the separator matrix. The only change is that $F(V_m)$ in Eq. 14 is replaced by $F_i(V_m)$ defined by Eq. 11 with $D_p = D_{p,\min}$ as given by Eq. 13.

Once the explicit dependence of (LOAD) on (FEED) has been found, the concentrations of the magnetic particles of interest at the outlet and inlet of the separator matrix, denoted by C_{out} and C_{in}, respectively, can be related by Eq. 31 of Part II as follows:

$$\frac{C_{out}}{C_{in}} = F(V_m), \qquad V_m = V_{m,\min} \qquad (17)$$

$$= \int_{W_o}^{100}\left\{1 - \exp\left[-\left(\frac{D_p}{m}\right)^n\right]\right\}dW + \frac{W_o}{100}, \qquad D_p = D_{p,\min} \qquad (18)$$

$$= \int_{W_o}^{100}\left\{1 - \exp\left[-\left(\frac{D_{p,\min}}{m}\right)^n\right]\right\}dW + \frac{W_o}{100} \qquad (19)$$

In particular, by substituting the explicit numerical values of (LOAD) at different values of (FEED), obtained from the preceding integration of Eq. 16, into Eq. 13 for $D_{p,\min}$, the above equation can be numerically integrated with respect to W to give the ratios C_{out}/C_{in} at different values of (FEED).

Comparison of Experimental Results with Model Predictions. Figures 5a and 5b show the typical comparison between the theoretical and experimental concentrations ratios, C_{out}/C_{in}, of the pulverized Illinois No. 6 coal particles in the beneficiated product obtained at a field intensity H_o = 2 Tesla (1.591 MA/m or 20 kOe) and at different total feed amounts per unit cross-section of the separator matrix. In each figure, the solid curve represents the theoretical prediction based on the numerical integration of the modeling Eqs. 13, 16 and 19 using the magnetic velocity distribution for the pulverized Illinois No. 6 coal at the superficial slurry velocity V_o given in the figure. The comparison indicates the concentration ratios of the pulverized Illinois No. 6 coal particles predicted by the model are in a reasonable agreement with experimental results.

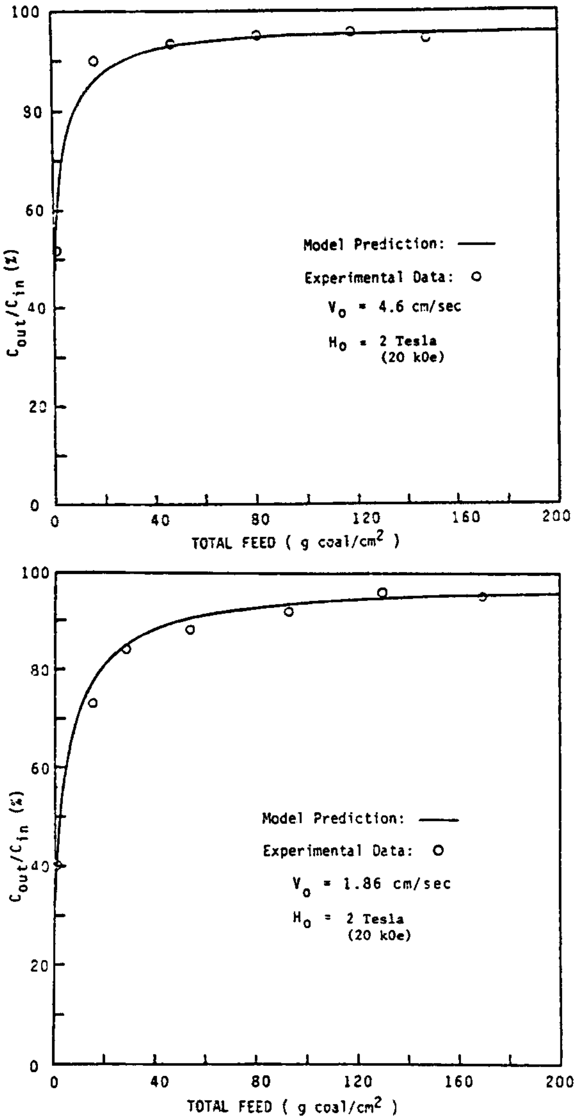

Figure 5. Comparison between theoretical and experimental concentration ratios of the pulverized Illinois No. 6 coal in the beneficiated product as a function of the total feed amount at (a) V_0 = 4.6 cm/sec (top), and (b) V_0 = 1.86 cm/sec (bottom).

Figures 6a and 6b illustrate the typical comparison between theoretical and experimental concentration breakthrough curves for the pulverized Illinois No. 6 coal, corresponding to the same feed characteristics and separation conditions as in Figures 5a and 5b. The comparison shows that the model accurately predicts the changes in ash and total sulfur contents of the beneficiated product with total feed amount for the pulverized Illinois No. 6 coal.

Computer Program for Implementing New Practical Model for HGMS Applied to Coal and Mineral Beneficiation

The procedure illustrated in the preceding application of the new practical model for predicting the technical performance of HGMS has been incorporated into a general FORTRAN computer program, entitled HGMS, developed during the course of this work (Oak, 1977). In particular, this computer program can be used to quantitatively predict the grade, recovery and separator capacity (concentration breakthrough) of HGMS applied to coal and mineral beneficiation. To use the program HGMS, the following input information related to the feed characteristic and separation conditions is needed.

1. Data of the float-and-sink characterization of the feed stream (for example, Table 1), including:
- Wt. % of each separated fraction
- Average density of each separated fraction, gm/cm^3
- Average magnetic susceptibility of each separated fraction, emu/gm

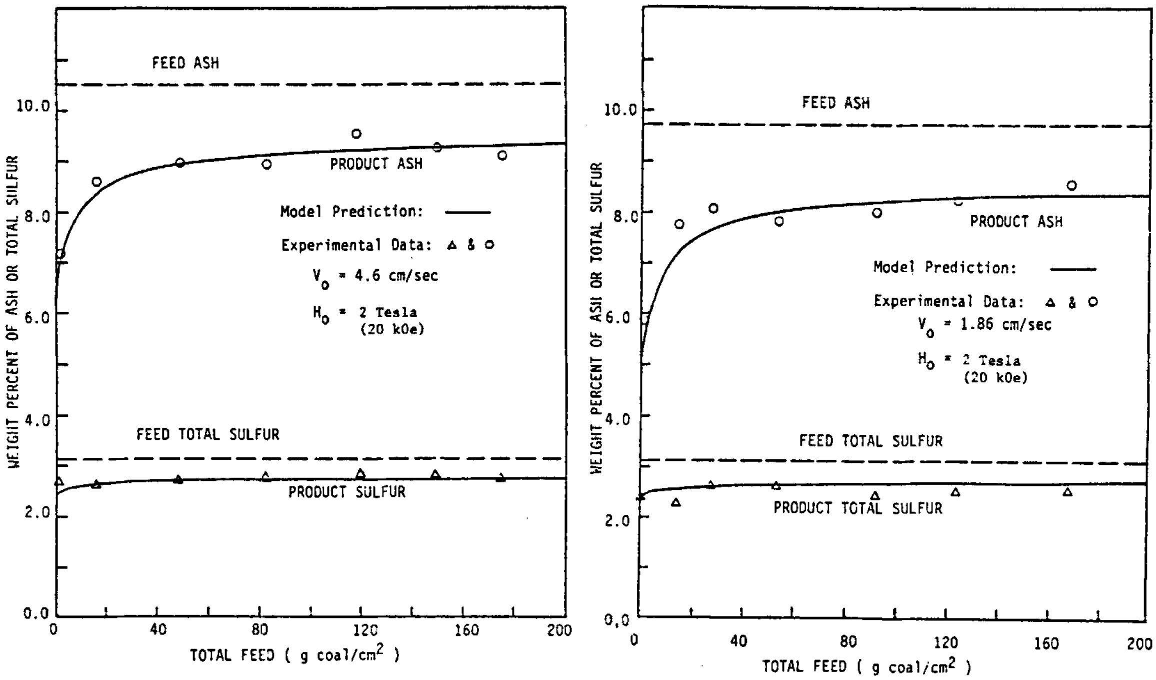

Figure 6. Comparison between theoretical and experimental ash and total sulfur concentration breakthrough curves of the pulverized Illinois No. 6 coal as a function of the total feed amount at (a) V_0 = 4.6 cm/sec (left) and (b) V_0 = 1.86 cm/sec (right).

- Concentrations of magnetic particles of interest in each separated fraction, such as the ash, total sulfur and pyrite sulfur contents in coal beneficiation, and the iron oxide content in iron ore beneficiation, wt. %.

2. Data of the particle-size distribution of the feed stream (for example, Figure 3a), particularly the particle diameter vs. the cumulative undersize or oversize wt. %.
3. Selected separation conditions, including:

- Length of the separator matrix, L (m)
- Intensity of the external magnetic field, H_o (Ma/m)
- Magnetization of the separator matrix packing wire, M (Ma/m)
- Average radius of the separator matrix packing wire, a (m)
- Packed fraction of the separator matrix packing, F (dimensionless)
- Apparent density of the particle buildup, d (gm/cm^3)
- Viscosity of the carrier fluid, η (Pa·s)
- Superficial slurry velocity, V_o (m/s)

When applying the program HGMS to coal beneficiation, the following output information is printed:

- Name of the coal used
- Data of float-and-sink characterization of the feed coal
- Data of particle-size distribution of the feed coal
- Separation conditions used
- Distributions of the ratio of the magnetic velocity to the fluid velocity
- Predicted grade and recovery of the pulverized coal along with its ash, total sulfur and pyrite sulfur contents in the beneficated product as a function of the total feed amount per unit cross-section of the separator matrix

Experimental Verification and Applications of Practical HGMS Model: Pulverized Pennsylvania Coals

The new practical model has also been successfully used to predict satisfactorily the grade, recovery and separator capacity (concentration breakthrough) observed in pilot-scale experimental studies of HGMS ap-

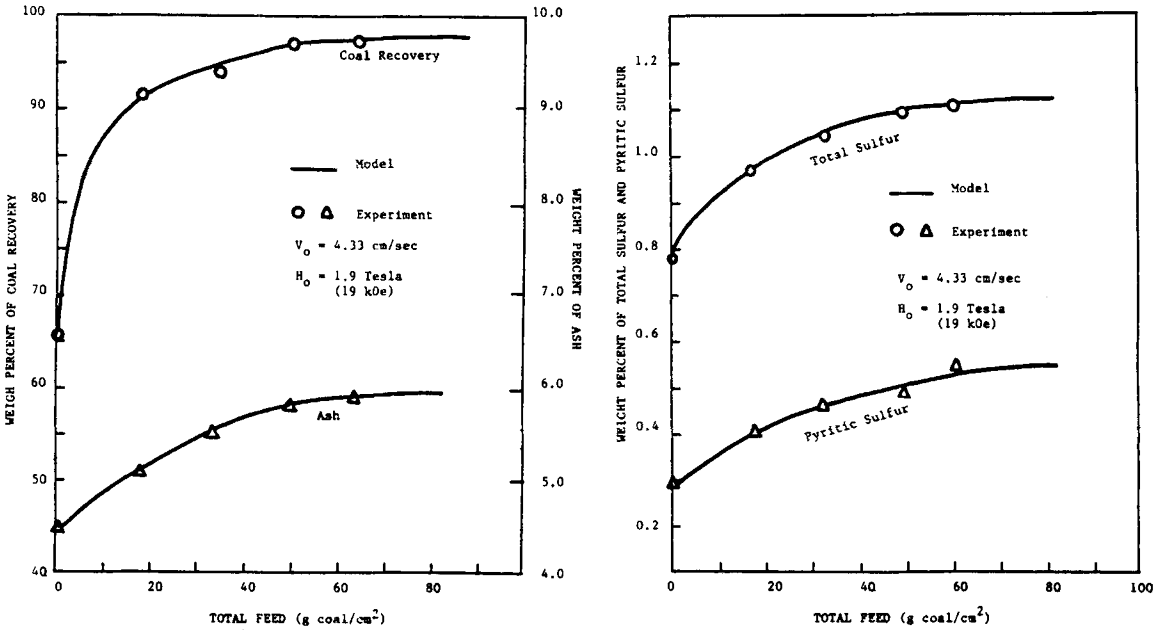

Figure 7. Comparison between theoretical and experimental grade and recovery of the pulverized Pennsylvania Upper Freeport coal as a function of the total feed amount: (a) coal recovery and ash content (left), and (b) total sulfur and pyritic sulfur contents (right).

plied to the beneficiation of water slurries of three pulverized Pennsylvania coals. For example, Table 1 illustrates the data of the float-and-sink characterization of the pulverized Pennsylvania Upper Freeport coal used; Figure 3b shows a log-log plot of the measured cumulative oversize weight percent distributions of the three pulverized Pennsylvania coals used. Figures 7a and 7b illustrate the typical comparison between the predicted and experimental concentration breakthrough curves for the pulverized Upper Freeport coal and its ash, total sulfur and pyritic sulfur contents obtained at a field intensity of 1.9 Tesla (1.511 MA/m or 19 kOe) and a superficial fluid velocity of 4.33 cm/s. Based on the close agreement between theory and experiment demonstrated in Figures 5a to 7b and the successful application of the model to other pulverized coals presented elsewhere (Oak, 1977), it can be concluded that there is now an experimentally verified, practical model available for predicting the technical performance of HGMS for the beneficiation of water slurries of pulverized coals prior to actual separation testing.

PRACTICAL APPLICATIONS OF MODELING RESULTS

The experimentally verified model, Eq. 30 to 35 of Part II, can be used to quantitatively predict the effects of separation variables such as flow velocity and field intensity on the technical performance of pilot-scale or industrial HGMS applied to coal beneficiation. As an illustration, the reductions in ash, total sulfur and pyritic sulfur as a function of the total recovery of clean coal in the magnetic beneficiation of a water slurry of pulverized Illinois No. 6 coal predicted by the model at a field intensity of H_o = 2 Tesla (1.591 MA/m or 20 kOe) and low superficial slurry velocities V_o = 0.23 and 4.6 cm/s, with other separation conditions being identical to those employed experimentally previously for Figures 5a to 6b. The results are shown in Figure 8a, which indicates that to obtain a beneficiated coal of low-sulfur and low-ash contents, the use of a low superficial slurry velocity is preferred. However, lower percentages of total recovery of the beneficiated coal are also observed for separations conducted at lower superficial slurry velocities. Thus, these results suggest that it will be particularly desirable to quantitatively determine the optimum range of superficial slurry velocity and other separation variables to optimize the magnetic removal of sulfur and ash, while achieving an economically acceptable recovery of the beneficiated coal. By using the new model developed in Part II, such optimum separation conditions can be quantitatively identified rather easily (Oak, 1977).

The new model can also be used to assess the technical and economical feasibility of the magnetic beneficiation of coal without extensive trail-and-error testing. For example, Figure 8b shows the predicted total recovery as well as reduction in ash and total sulfur as a function of the total feed amount per unit cross-sectional area of the separator matrix corresponding to the case with V_o = 4.6 cm/s previously shown theoretically and experimentally as illustrated in Figure 8. The predicted instantaneous changes in ash and total sulfur can be used to find the processing throughput after which an HGMS unit is loaded. Along with the total recovery curve predicted in the figure, one can easily determine the proper separation duty cycle under the selected process conditions. Thus, the new model developed can be used to provide the needed information for the engineering design and cost estimation of wet HGMS processes applied to coal beneficiation (Singh, 1982).

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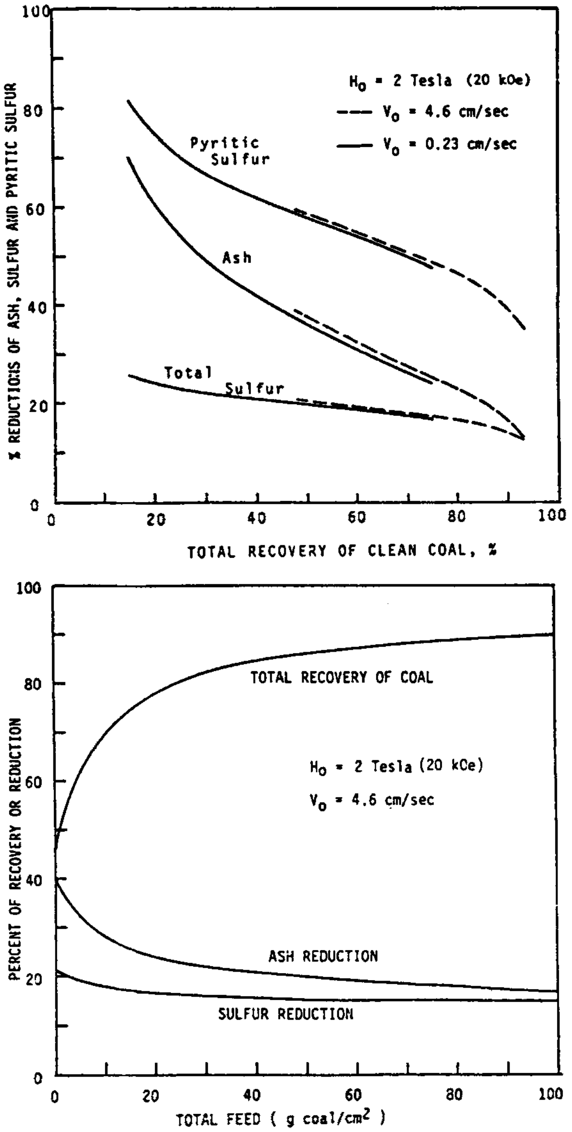

Figure 8. Applications of the practical model to the pulverized Illinois No. 6 coal: (a) predicted reductions in ash, total sulfur and pyritic sulfur contents as a function of the total recovery of the beneficiated coal (top), and (b) predicted coal recovery and reductions in ash and total sulfur as a function of the total feed amount (bottom).

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NOTATION

a = average radius of particle collecting wire, m

A_{io} = average ash, sulfur and pyritic sulfur contents (i = 1, 2 and 3, respectively) of the pulverized feed coal at a cumulative weight fraction W_o with negative magnetic susceptibilities, wt. %

$A_i(W)$ = average ash, sulfur and pyritic sulfur contents (i = 1, 2 and 3, respectively) of the pulverized feed coal at a cumulative weight percent W, wt. %

b = radius of the cylindrical particle buildup, m

b_a = dimensionless particle buildup radius, $b_a = b/a$

C = concentration of magnetic particles, kg/m^3

C_{in} = concentration of magnetic particles at the inlet of the separator, kg/m^3

C_{out} = concentration of magnetic particles at the outlet of the separator, kg/m^3

d = apparent density of the particle buildup, kg/m^3

D_p = diameter of particle, m

f = relative buildup volume on the upstream side of the wire, dimensionless

f_{max} = maximum relative buildup volume on the upstream side of the wire, dimensionless

F = packed fraction of the separator matrix, dimensionless

F_D = fluid drag force acting on the particle, N

$F_{D\theta}$ = angular component of the fluid drag force, N

F_M = magnetic force acting on the particle, N

$F_{M\theta}$ = angular component of the magnetic force, N

FEED = total weight of feed particles per unit cross-section of the separator matrix, kg/m^2

$F(V_m)$ = magnetic velocity distribution of the feed stream

$F_i(V_m)$ = magnetic velocity distributions of ash, sulfur and pyritic sulfur (i = 1, 2 and 3, respectively) of the pulverized feed coal

g = acceleration of gravity, 9.8 m/s^2

H_o = external magnetic field intensity, Ampere turns per meter (A/m)

l = length of packing wire, m

L = length of the separator matrix, m

LOAD = total weight of captured magnetic particles per unit cross-section of the separator matrix, kg/m^2

M = magnetization of the particle collecting wire, A/m

r = radial distance of the particle from the origin, m

r_a = dimensionless radial coordinate of the particle, $r_a = r/a$

R = radius of the particle, m

R_c = critical entering coordinate of the particle, dimensionless

t = time variable, s

T_i = average ash, sulfur and pyritic sulfur contents (i = 1, 2 and 3, respectively) of the pulverized feed coal, wt. %

$U(R)$ = cumulative weight fraction of particles having a radius smaller than R

V_o = superficial fluid velocity in the separator matrix defined by V_∞/ϵ, m/s

V_m = magnetic velocity of the particle, m/s

$V_{m,min}$ = minimum magnetic velocity for particular buildup, m/s

V_∞ = free stream fluid velocity, m/s

W_o = cumulative weight percent of particles having negative magnetic susceptibilities

x = axial coordinate of the particle axial distance along the length of the separator matrix

y_a = reduced y coordinate defined by y/a

y_a^∞ = initial reduced y coordinate of the particle at $x = -\infty$

Greek Letters

η = viscosity of the carrier fluid, Pa·s

δ = average fluid boundary layer thickness outside the cylindrical particle buildup, m

ϵ = void fraction of the separator matrix

θ = angular coordinate of the particle with respect to the x-axis, or one half of the particle buildup angle

θ_c = critical buildup angle

θ_{max} = maximum buildup angle

μ_o = permeability of the free space, 4×10^{-7} Henry/m

τ = modified time variable, s

ρ_f = density of the carrier fluid, kg/m^3

ρ_p = density of the particle, kg/m^3

χ = magnetic susceptibility of the particle, dimensionless

$\chi(W)$ = magnetic susceptibility of the pulverized feed coal at a cumulative wt. %, W

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One Variety of Quick Estimation Schemes for Parameters in Kinetic Models

Quick estimation of the parameters in a process model can play a significant role in kinetic studies. This work is aimed at developing a quick estimation strategy by means of process discretization and evaluating errors engendered by the strategy. A systematic error evaluation method and guidelines to select the best discretization relation used in the strategy are described.

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SCOPE

We examine a number of aspects of estimating the coefficients in kinetic models that are not widely known. In particular we examine the class of methods in which the derivatives in the differential equations are approximated by a proper difference relation to yield a set of algebraic equations that can be solved for the coefficients. Such a procedure makes the estimation of the coefficients quite simple and is quicker than repeatedly integrating the differential equations themselves followed by least squares. But the discretization leads to an approximate model. To obtain reliable estimates of the coefficients one needs to know how the approximation incurred in the discretization influences the errors in the estimates. One also wants to control the size of the errors.

We describe how to develop a general and quick estimation strategy using commonly encountered discretization schemes and evaluate the errors in the coefficient estimates. We also provide guidelines as to how to select the best discretization scheme for a given estimation problem.

CONCLUSIONS AND SIGNIFICANCE

The analysis presented here leads to a straightforward interpretation of how errors in kinetic coefficients are engendered in quick estimation procedures in which differences are substituted for derivatives. The relative error in a discretization scheme was presented, and was utilized to check whether or not the coefficient estimates were reliable. Guidelines were cited as to how to select the best discretization scheme. From the results of this study, one can clearly determine the limitations of an estimation procedure and can select the best discretization scheme for a given estimation procedure.

INTRODUCTION

Estimation of unknown parameters in dynamic processes by means of quick and simple methods can make a significant contribution to adaptive process control (Landau, 1979) and process simulation as well as incipient process fault detection and diagnosis (Himmelblau, 1978). Such estimates can also be used to generate initial guesses of the unknown parameters for an optimal parameter seeking method (Glowinski and Stocki, 1981).

In general, parameter estimation techniques can be divided into three categories: (a) the extended Kalman filter (Jazwinski, 1970); (b) iterative methods in which iterative numerical integration of a set of ordinary differential equations is followed by an iterative optimization algorithm (Himmelblau et al., 1967; Froment, 1975; Pexidr, 1974; Seinfeld, 1970; Watanabe and Shimizu, 1975); and (c) noniterative methods in which a process model is discretized by a proper integration formula and then the set of resulting algebraic equations is solved simultaneously (Glowinski and Stocki, 1981).

The methods in categories (a) and (b) engender a heavy computational load, but if properly designed, can yield quite accurate

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